A genetic algorithm with embedded constraints

– An example on the design of robust D-stable IIR filters

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Outline

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- The stability problem of an IIR filter
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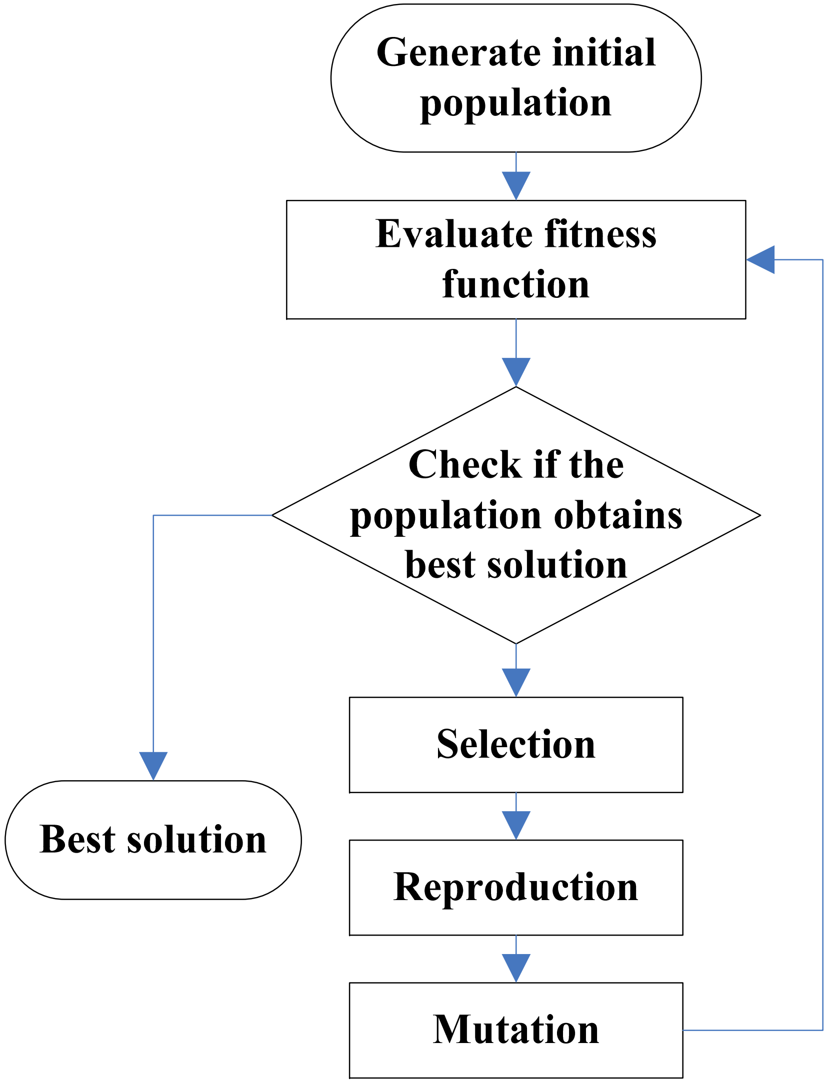
Motivation

- In general, the strategy of GA on an optimal problem with some constraints:
  - Test whether chromosomes satisfies the constraints
  - Drop and re-generate new chromosomes if the constraints are not satisfied
  - Calculate the corresponding fitness value if the constraints are satisfied
  - Evolution
- Is it possible that we can embed the constraints into the evolution of GA, so that it is not necessary to test the constraints?

Introduction on GA



Improved GA

- Improved Crossover

$$os_c^1 = [os_1^1 \quad os_2^1 \quad \ldots \quad os_{no\_vars}^1] = \frac{P_1 + P_2}{2}$$

$$os_c^2 = [os_1^2 \quad os_2^2 \quad \ldots \quad os_{no\_vars}^2] = P_{\max}(1-w) + \max(P_1, P_2)w$$

$$os_c^3 = [os_1^3 \quad os_2^3 \quad \ldots \quad os_{no\_vars}^3] = P_{\min}(1-w) + \min(P_1, P_2)w$$

$$os_c^4 = [os_1^4 \quad os_2^4 \quad \ldots \quad os_{no\_vars}^4] = \frac{(P_{\max} + P_{\min})(1-w)(P_1 + P_2)}{2}$$

$os_c^1 \sim os_c^4$: chromosomes of next generation

P_1, P_2 : the parents

The stability problem of an IIR filter

- the transfer function of a IIR filter be described as

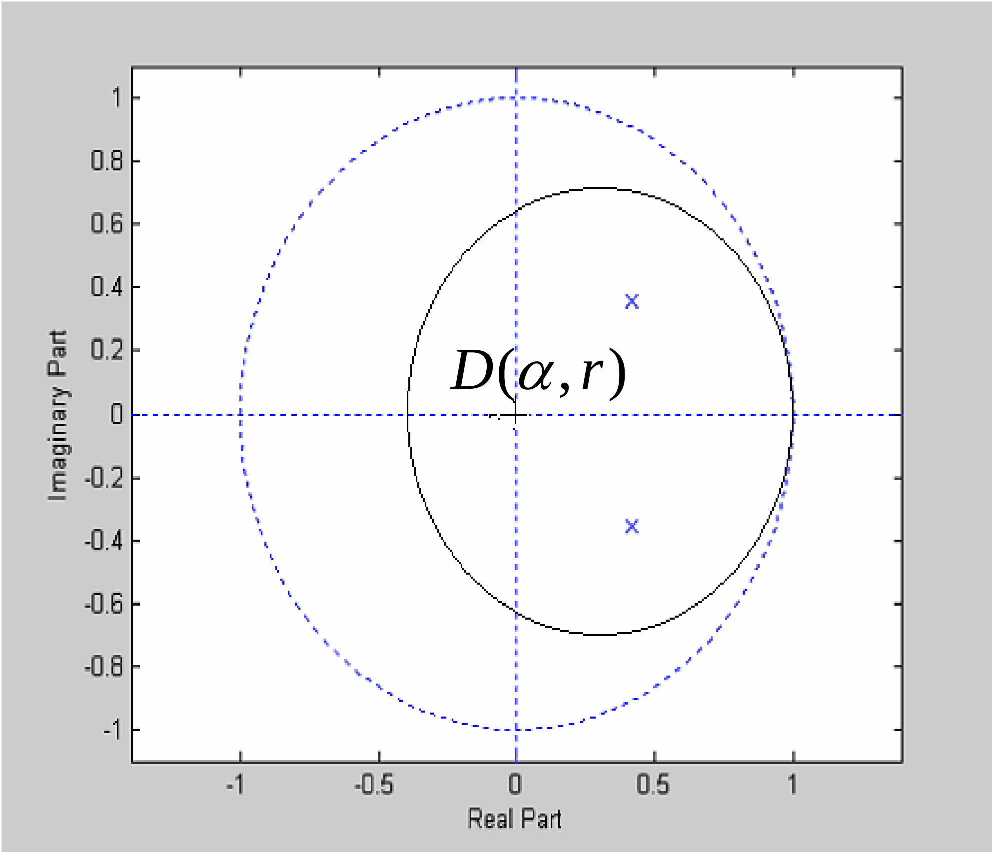
$$H(z) = \frac{\sum_{k=0}^{m} b_k z^{-k}}{\sum_{k=0}^{n} a_k z^{-k}}$$

where

a_k and b_k are parameters to be designed.

The stability problem of an IIR filter

- $H(z)$ is stable if all poles lie inside unit disk
- $H(z)$ is D-stable if all poles lie inside the disk $D(\alpha, r)$ contained in unit disk



The stability criterion of IIR filter

Theorem 1:

Let $a(z) \equiv \sum_{k=0}^{n} a_k z^{-k}$.

If the following inequality is satisfied

$$\left| \sum_{k=1}^{n} \frac{a_k}{a_0} (re^{-j\theta} + \alpha)^{-k} \right| < 1 \qquad \forall \theta \in [0, 2\pi] \tag{1}$$

then the solutions of $a(z) = 0$, will lie inside the disk $D(\alpha, r)$ with $|\alpha| + r < 1$ and $|\alpha| \le r$.

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The stability criterion of IIR filter

Theorem 2:

Two polynomials: $d(z) \equiv \sum_{k=0}^{n} a_k z^{-k} \qquad \hat{d}(z) \equiv \sum_{k=0}^{n} \hat{a}_k z^{-k}$

both satisfy (1)

Then polynomial:

$$c(z) = \left[sign(a_0 \hat{a}_0) \lambda a_0 + (1-\lambda)\hat{a}_0 \right] + \sum_{k=1}^{n} \left[\lambda a_k + (1-\lambda)\hat{a}_k \right] z^{-k}$$

satisfy (1)

$$sign(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases} \qquad 0 \leq \lambda \leq 1$$

The stability criterion of IIR filter

Theorem 3

Let $u_{b1} = \dfrac{|a_0|}{l}$ and $u_{bi} = \dfrac{1}{l}(u_{b(i-1)} - |a_{i-1}|)$ for $i = 2, 3, \cdots, n,$

Then the polynomial $d(z) = \sum_{k=0}^{n} a_k z^{-k}$ with $|a_i| < u_{bi}$

satisfy (1) for $i = 2, 3, \cdots, n,$ and any real number a_0

$$\boxed{l = (r - |\alpha|)^{-1}}$$

How to embed the constraints (stability criterion) into evolution of GA?

- **Corollary 1:**

Suppose that the two chromosomes:

$$d \equiv (a_0 \quad a_1 \quad a_2 \quad \cdots \quad a_n) \quad \hat{d} \equiv (\hat{a}_0 \quad \hat{a}_1 \quad \hat{a}_2 \quad \cdots \quad \hat{a}_n)$$

satisfy the inequality (1).

The chromosome in the offspring (by crossover)

$$c \equiv \left(sign(a_0\hat{a}_0)\lambda a_0 + (1-\lambda)\hat{a}_0 \quad \lambda a_1 + (1-\lambda)\hat{a}_1 \quad \lambda a_2 + (1-\lambda)\hat{a}_2 \quad \cdots \quad \lambda a_n + (1-\lambda)\hat{a}_n\right),$$

satisfy the inequality (1).

How to embed the constraints (stability criterion) into evolution of GA?

- **Corollary 2:** Suppose that the chromosome $d = \begin{pmatrix} a_0 & a_1 & a_2 & \cdots & a_n \end{pmatrix}$ satisfy (1). The new chromosome $m_d \equiv \begin{pmatrix} m_{a_0} & m_{a_1} & m_{a_2} & \cdots & m_{a_n} \end{pmatrix}$ (the result of the mutation from d satisfy (1) if the mutation is perform according to Theorem 3.

The mutation

Step 1. let $m_{a0} = a_0$

Step 2. random $|m_{a_1}| < \frac{|m_{a_0}|}{l} = u_{b1}$

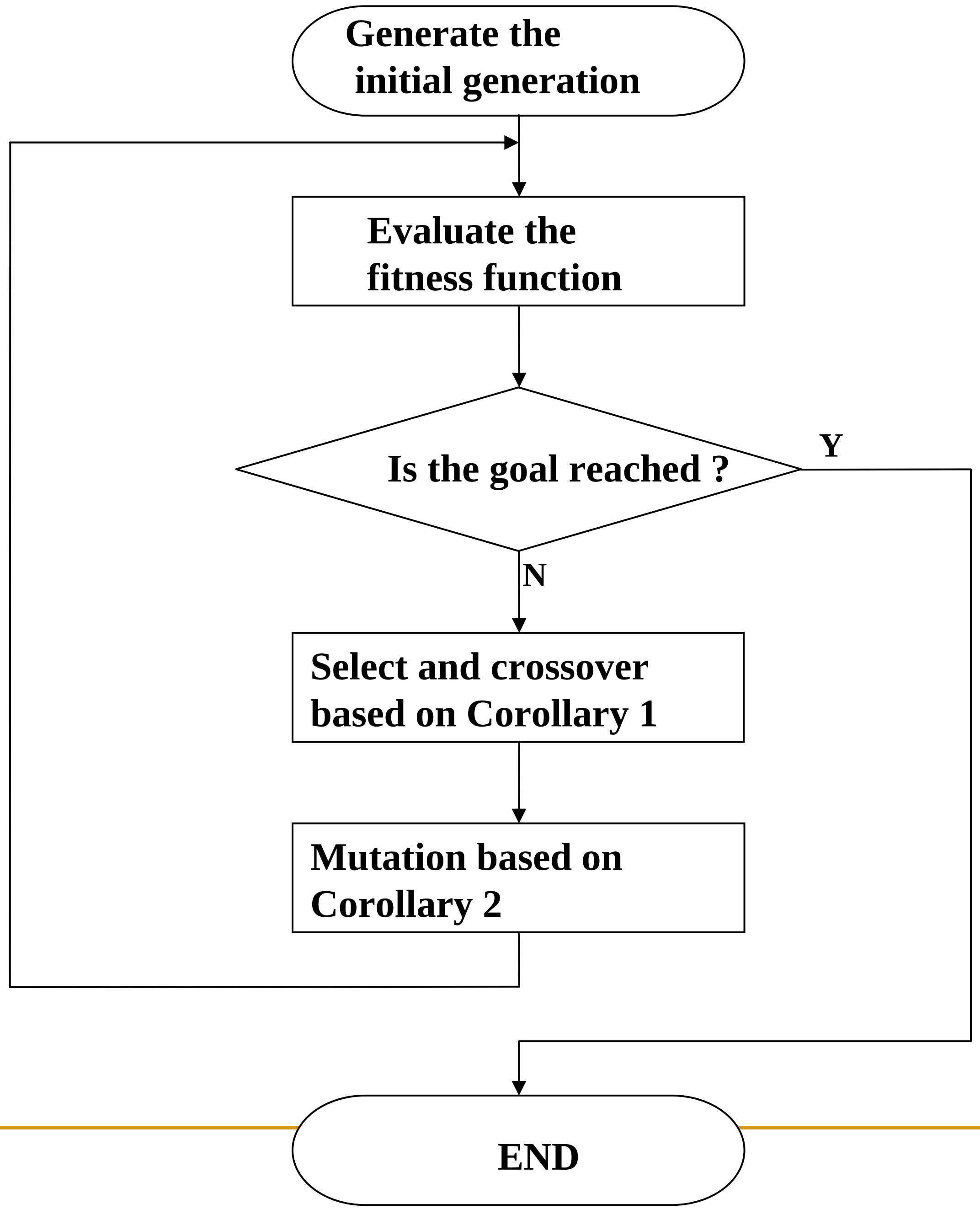
Step 3. random $|m_{a_2}| < \frac{1}{l}(u_{b1} - |m_{a_1}|) = u_{b2}$

Step 4. random $|m_{a_3}| < \frac{1}{l}(u_{b2} - |m_{a_2}|) = u_{b3}$

$\vdots$

Step $(n+1)$. random $|m_{a_n}| < \frac{1}{l}(u_{b(n-1)} - |m_{a_{n-1}}|) = u_{bn}$

How to embed the constraints (stability criterion) into evolution of GA?

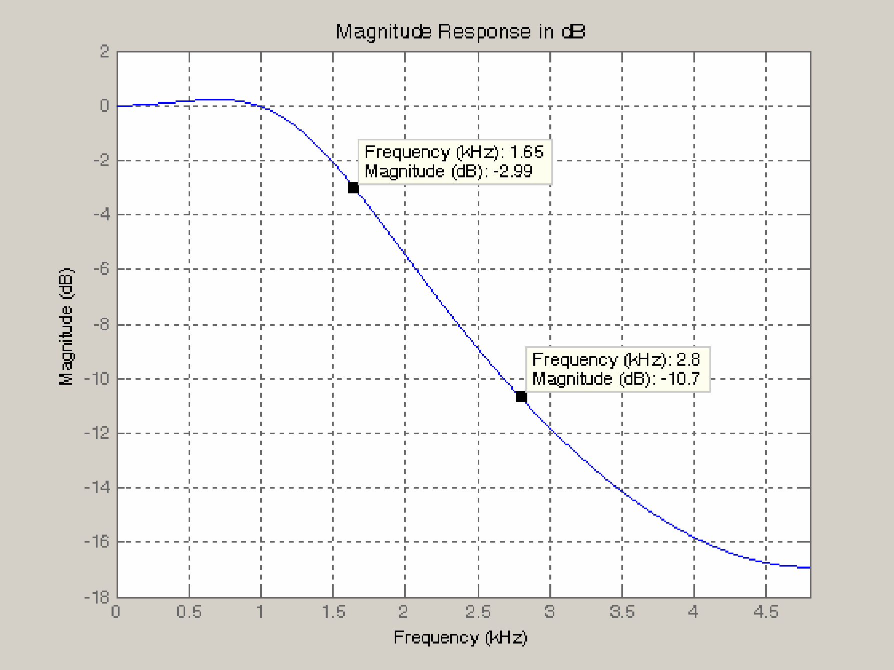


Simulation results

- **A low-pass IIR filter with two poles**

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

The target frequency response



Simulation results

- The Goal:
  - Design a_k and b_k so that
    - Minimize the MSE between designed frequency response and target frequency
    - The filter is D(0.3,0.7)-stable

Simulation results

- The fitness function is defined as

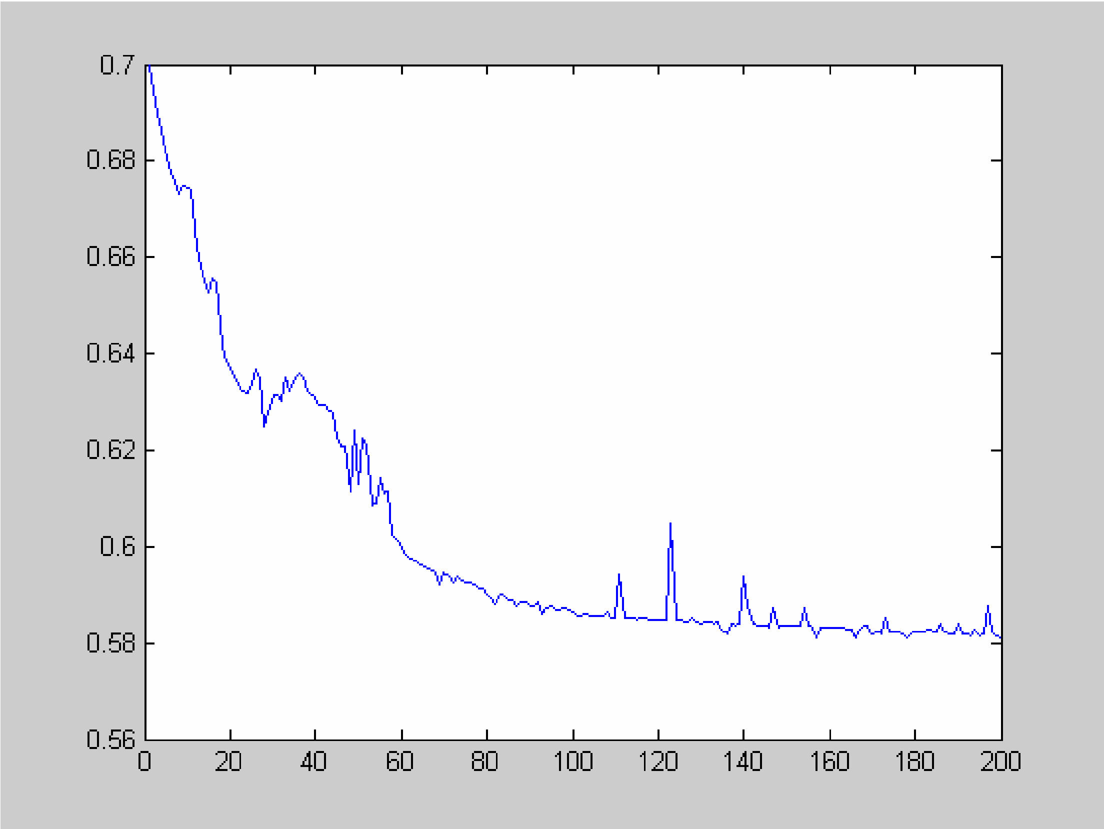
$$fitness(p) = \frac{1}{E_p^{\ 2} + 1}$$

in which

$$E_p \equiv \frac{\sum_{\Omega} \left| H_p(\Omega) - H_I(\Omega) \right|^2}{N_s}$$

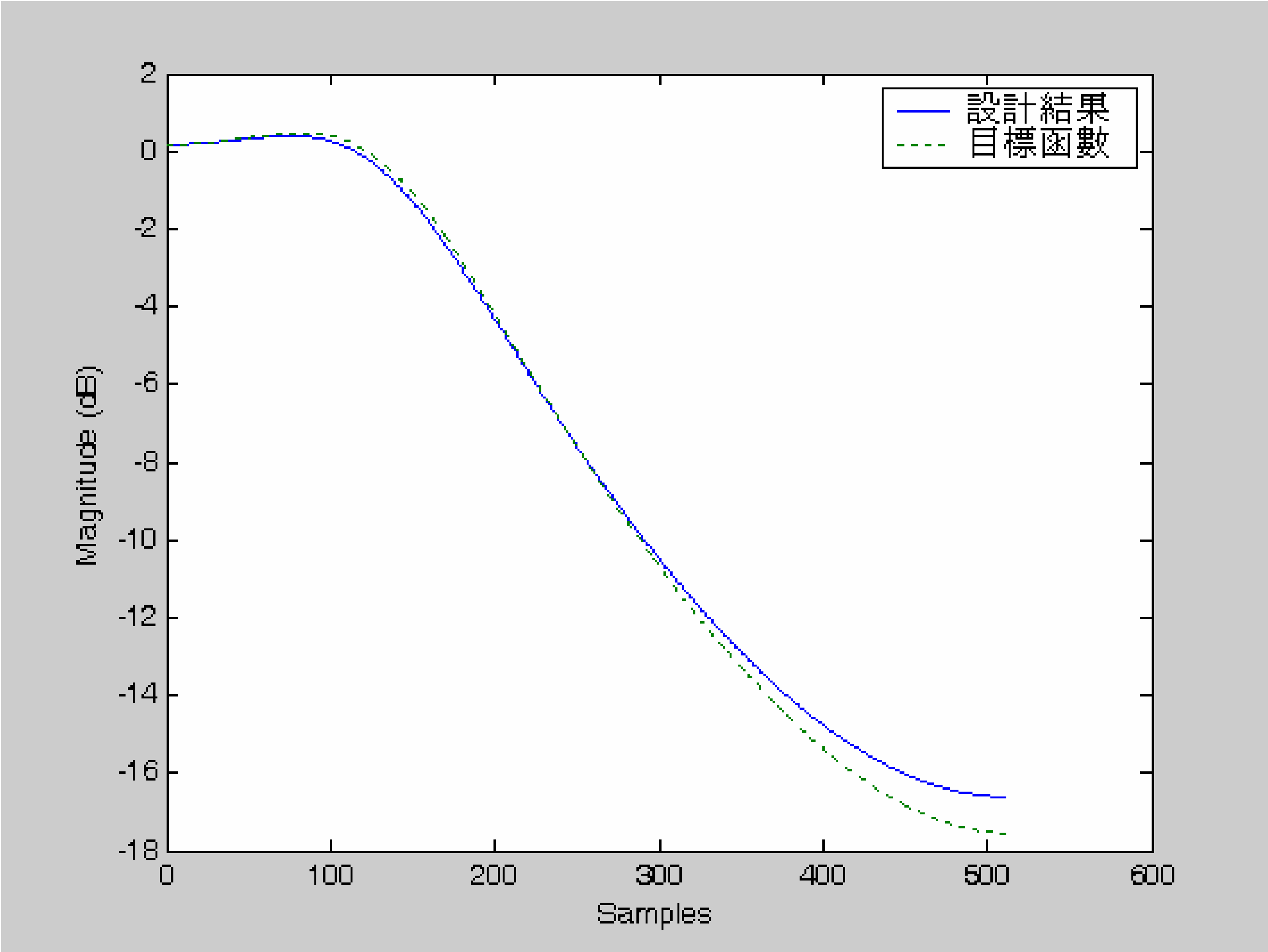
$$H_p(\Omega) = \frac{\sum_{k=0}^{m} b_k^{\,p} e^{-jk\Omega}}{1 + \sum_{k=1}^{n} a_k^{\,p} e^{-jk\Omega}}$$

Simulation results



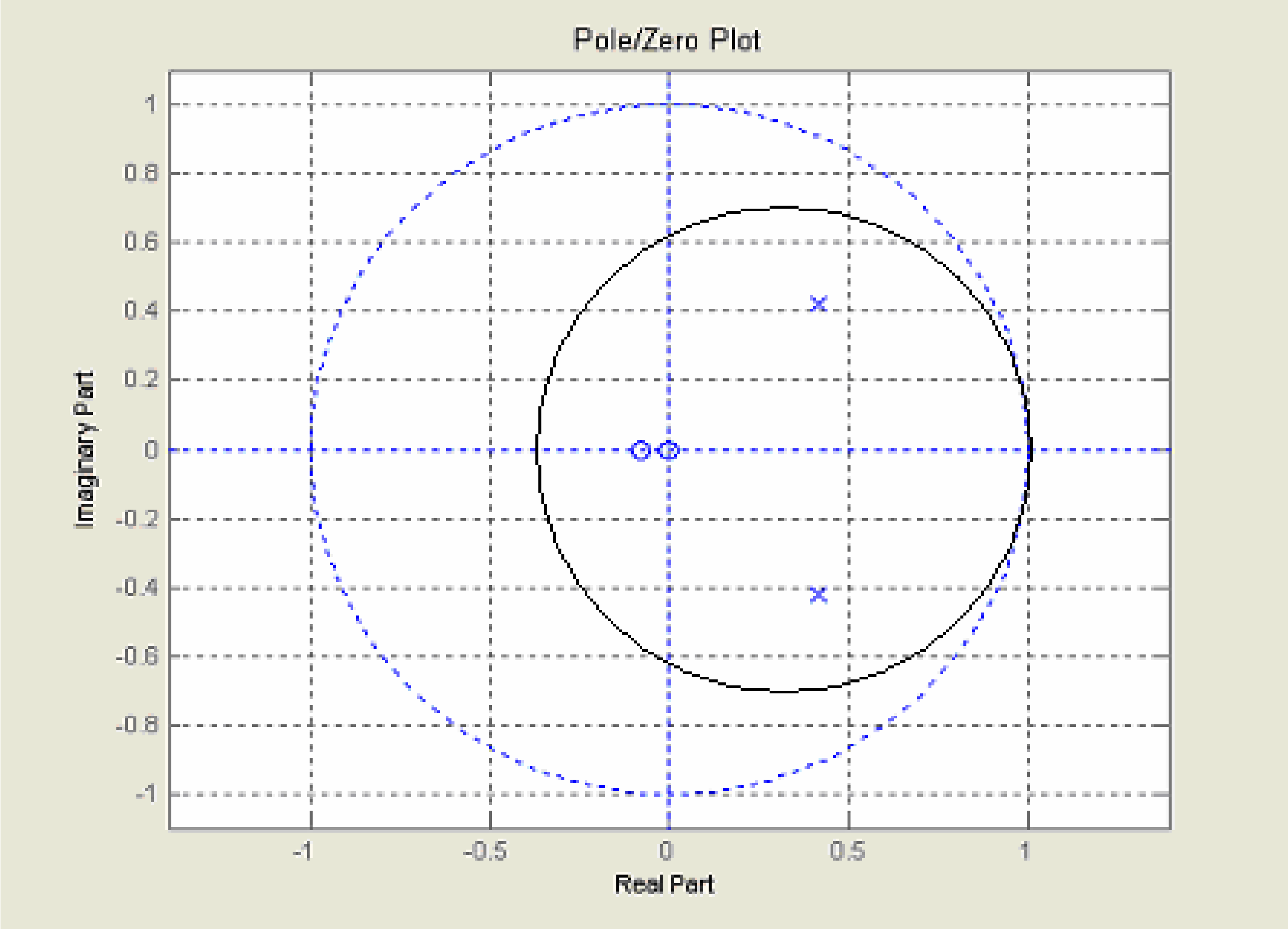
The maximum distance between the poles as well as zeros and the center α =0.3+j0

Simulation results



Simulation results

The poles and zero of designed IIR filter



Discussions

- The performance is better than those of the algorithm by using trial and error
- The stability criterion (constraint) is a sufficient condition and also the proposed crossover and mutation is – condense the search space
  - A less conservative criterion (algorithm) should be found
- Not all optimal problem can be applied by the proposed algorithm, since the embedded evolution is not easy to find

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Q&A

- Thank you for listening