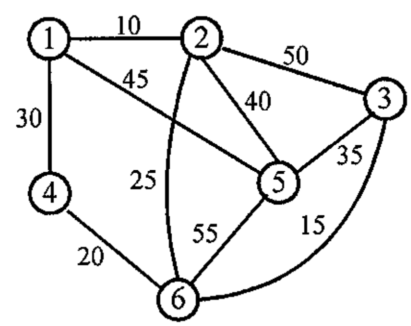
Dept. of Computer Science and Engineering, National Sun Yat-sen Univ.

2010 (July) PhD Qualifying Exam.   Computer Algorithms

1. [20%] Find the minimum cost spanning tree in the following graph via Kruskal's algorithm. You should show how to get the answer.



2. [20%] Given a set S of n points in the plane, please design a Divide-and-Conquer algorithm to solve the 2-dimensional closest pair problem to output the distance between two closest points.

3. [20%] Let p be an m-bit prime shared by A and B, and they keep p confidential. Please design a randomized algorithm for interactive proofs based on quadratic residues (QR) such that B can be convinced that A is the real A with probability $1\text{-}2^{-m}$ after performing the interactive algorithm successfully.

4. [20%] Let A be a machine that can solve $f(x) = 0$, i.e., A can output all possible values of x such that $f(x) = 0$. Assume that A can solve $f(x) = 0$ for any polynomial $f(x)$ with degree less than n, but cannot solve $f(x) = 0$ if $f(x)$ is with degree not less than n, where n is an integer greater than 3. Please construct a machine B using A such that B can solve $g(x)(h(x)+k(x)) = 0$ where $g(x)$, $h(x)$, and $k(x)$ are three polynomials with degree n-1, n-2, and n-3, respectively.

5. [20%] There are n items $a_1, a_2, \ldots, a_n$ with $0 < \text{Size}(a_i) \le 1$ and $1 \le i \le n$. The bin packing problem is to determine the minimum number of bins of unit capacity, i.e., the capacity of each bin is 1, to accommodate all n items. The problem is NP-hard. Please design a polynomial-time approximation algorithm for the bin packing problem and prove that the number of bins used in your algorithm is at most twice of the optimal solution.