國立中山大學資訊工程學系
105 學年度第 1 學期博士班資格考試

科目：演算法

Algorithms, 2017/01/16

1. (10) Define the classes of problems *P*, *NP*, and *co-NP*. Define *polynomial-time reduction* and *log-space reduction*. Define the class of problems *NP*-complete. Show that a polynomial-time algorithm for any *NP*-complete problem implies *P*=*NP*.

2. (20) Let $a_1, a_2, \ldots, a_n$ be a sequence of n distinct integers in ascending order. Given an integer x in the list, binary search can be used to determine index k such that $a_k = x$ in $O(\log n)$ time. Suppose that k is much smaller than n in some application. Design an algorithm to search the number x in $O(\log k)$ time. Note that the value of k is unknown in advance.

3. (20) Show that n integers in the range $[1, n^2]$ can be sorted in $O(n)$ time by giving a linear time algorithm, or prove that it cannot be done in $O(n)$ time. What can be concluded if the n numbers are in the range $[1, n^3]$, $[1, n^4]$, $\cdots$, $[1, n^k]$ for some fix integer k.

4. (20) Let $X = x_1 x_2 \cdots x_m$ be a long sequence of integers. Assume that $0 < x_i < n$ for $1 \le i \le m$. It is also known that some element x_j appears at least $\lceil (n+1)/2 \rceil$ times. Design an algorithm to read the long sequence X only once, and find out the majority element x_j in $O(\log n + \log m)$ space (in term of number of bits), or show that this is impossible. Note that to store an element in X needs $\log n$ bits, and to store the number of times an element appears needs $\log m$ bits.

5. (30) We are going to design an approximation algorithm for the traveling salesman problem. The input to the problem is a weighted complete graph $G = (V, E, w)$ where w is a positive weight function $w : E \to R^+$. Assume that w satisfies *triangle inequality*, i. e. $w(x, y) + w(y, z) \ge w(x, z)$ for all $x, y, z \in V$. The output of your algorithm should be a *good* solution C, which is a spanning cycle of G.

   (a) Let T be a spanning tree of G, and $v_1, v_2, \ldots, v_n$ be the depth-first traversal of T starting from some vertex v_1. Let $C = v_1, v_2, \ldots, v_n, v_1$ be a spanning cycle of G which is constructed based on the spanning tree T. Show that the total distance of C, $w(C)$, is bounded by $2w(T)$. (10)

   (b) Design an approximation algorithm for computing a spanning cycle C of G with $w(C) \le 2w(C^*)$, where C^* is the optimal solution. (20)