Dept. of Computer Science and Engineering, National Sun Yat-sen Univ.
First Semester of 2010 PhD Qualifying Exam Computer Algorithms

1. Design an algorithm for solving the *shortest path* problem on a graph. What is the time complexity of your algorithm? Why? (10%)

2. The *merging* problem is to merge two sorted lists into one sorted list. Prove that the lower of comparisons required for the merging problem of two lists (both with n elements) is $2n-1$. (15%)

3. In the solution searching strategy, there are two basic ways to visit solution nodes: *depth-first search* and *breadth-first search.*

   (a) What data structure should be used in depth-first search? Why? (5%)

   (b) What data structure should be used in breadth-first search? Why? (5%)

   (c) What is the *best-first search* scheme? (5%)

4. (a) The *selection* problem is to select the kth smallest element among n input elements. Please design an algorithm to solve the problem. The time required for your algorithm should be $O(n)$. (Hints: The *prune-and-search* strategy is a good approach.) (10%)

   (b) What is the general recurrence form for computing the time complexity of a prune-and-search algorithm (not particular for the selection problem)? (5%)

5. Prove that the *clique* decision problem polynomially reduces to the *node cover* decision problem. (15%)

6. There is a recursive C function to find the maximum of n elements stored in an array:

```
max(int a[ ], int n)
/* The data elements are stored in a[1],a[2],...,a[n] */
/* n: number of elements in array a[ ] */
{
   printf("ENTERING\n");
   if(n==1)
      return(a[1]);
   else
      if (a[n]>=max(a,n-1))
         return(a[n]);
      else
         return(max(a,n-1))
}
```

In the main program, we have a call $max(a, m)$, $m \geq 1$, to the above function.

(a) How many lines of ENTERING are printed at least? What situation will cause the least number of message ENTERING to be printed? (3%)

(b) How many lines of ENTERING are printed at most? What situation will cause the most number of message ENTERING to be printed? (4%)

(c) How many lines of ENTERING are printed in average? It is assumed that the data elements are in a uniform distribution. And, in this problem, you need only to write down the recurrence formula, need not derive the formula. (4%)

(d) Rewrite the function so that the function is a more efficient recursive maximum finder. (4%)

7. Let B_m and G_m denote the binary and *Gray code* representations of a positive integer m, respectively, where B_m and G_m are binary bit strings. Let $B_m = b_{n-1}\cdots b_2 b_1 b_0$ and and $G_m = g_{n-1}\cdots g_2 g_1 g_0$, where each b_i or g_i is of value either 0 or 1. The conversion from the binary representation to the Gray code representation is given by $g_k = b_{k+1} \oplus b_k, 0 \le k \le n-1$, where $b_n = 0$.

(a) Give the conversion method from the Gray code representation to the binary representation. (5%)

(b) Prove the following: The Gray code representations of two binary numbers x and $x \oplus 2^i$, $1 \le i \le n-1$, differ exactly at two bit positions. (10%)